Math 7640 Numerical Linear Algebra Spring 2018
Project 1: Singular Value Decomposition

Part I: Image Processing[1]

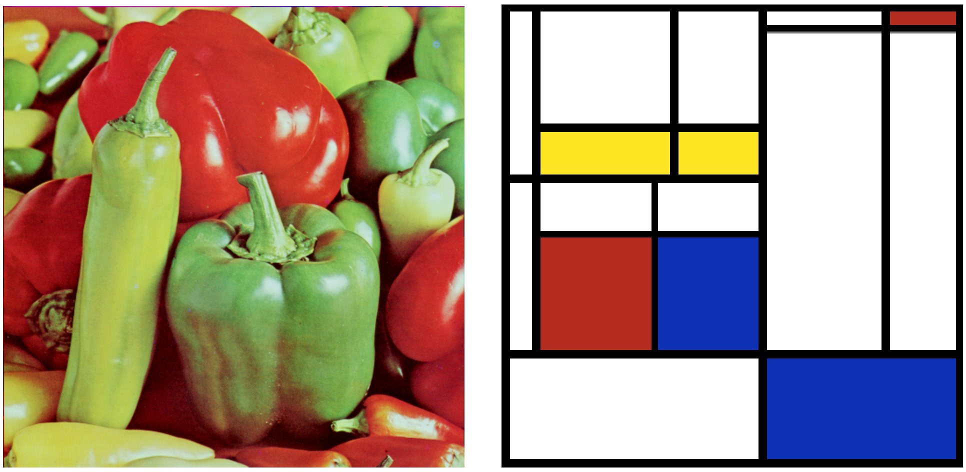

Figure 1: Left: Peppers image common SVD example. Right: Piet Mondrian inspired painting.

1. Using the grayscale version of the images:
(i). Perform Singular Value Decomposition on the peppers image.
(ii). Plot the singular values to see where the information content begins to wane.
(iii). Choose a set of singular values $\sigma_1, \ldots, \sigma_k$, which you think will demonstrate the tapering off of the quality of the image.
(iv). Plot pictures and note the error.

2. Look at the Mondrian painting, picture Mondrian.jpg. Do you think that this will require bigger or smaller k? Go ahead and run the program on it. What explains the difference?

3. from above we see that we can approximate each greyscale matrix A, with A_k, a sum of rank-one matrices. A requires nm entries to store, how many does A_k? For the peppers example, evaluate the ratio in the number of entries that need to be stored for A_k to A for $k = 5, 10, 20, 40, 80$. Since this ratio shouldnt exceed 1, we have an upper bound on k. Express it in terms of of n and m and evaluate it for the peppers example.

[1] compilation of homework assignments and pictures found online at http://wpressutexas.net/coursewiki/images/0/0d/Svd-exercise.pdf

Part II: Principal Component Analysis[2]

In the course website find the file nutrition data: the data table lists for each of 16 european countries the relative consumption of certain food items, such as tea, jam, coffee, yoghurt, and others.

1. Perform Principal Component Analysis on this data and find the first 2 Principal Components: PC1 and PC2.

2. Plot a loadings plot of PC1 against PC2. Which are the important variables in the first component? Do the results suggest any correlations of variables? And the second component?

3. Visualize the data with respect to the first 2 principal components. Can you interpret the results?

[2]compilation of material found online at https://learnche.org/pid/latent-variable-modelling/principal-component-analysis/pca-exercises